SHAKE-BOX TESTS

Holger Wienbroer¹, Daniel Rebstock¹, Gerhard Huber¹

ABSTRACT

Non-linear constitutive relations for saturated soils under earthquake like excitation have to be validated by laboratory tests. These laboratory tests can be performed by element tests and shake-box tests at 1-g or n-g (centrifuge tests). The tests presented here refer to shake-box tests at 1g with a new smooth boundary condition due to pivoted frames. Beside the technical realisation of the shake-table, the construction of the laminar shear box is described also. In general tests with two different heights of the shake-box are feasible. The instrumentation includes horizontal displacements of all frames (contour of boundary), settlement and pore water pressure. Requirements and interpretation related to this instrumentation are presented.

Experiments under fully saturated conditions were performed with a homogenous soil column using middle grained quartz sand. Usually measured or synthesized earthquake signals are used for base excitation. For a systematic investigation sinusoidal excitation is advantageous. In this case appropriate values of frequency and displacement amplitude for optimal observability had to be found. Therefore variations of both were performed. The state of soil during the test (void ratio and effective stress) and pore water pressure is transient, but in contrast to real earthquake signals, they can reach a stationary state. Stationary behaviour for higher number of cycles (attractors) was found for e.g. pore pressure or envelope of displacements. The influence of the state of soil under various loading conditions could be identified clearly.

INTRODUCTION

The type and state of soils near the surface (down to about 100m) have major influence on damages of structures build on them. It can be shown that waves propagating from the seismic source tend to propagate as horizontal shear waves towards the surface (Kramer, 1996). During real earthquakes most damages result from these shear waves. In many cases the soil behaviour near the surface is therefore considered in simulations as a 1-D-wave-propagation. This simplification is acceptable if the considered soil region is far away from the seismic energy source, then a periodic boundary condition is justified (cf. Fig. 1).

The non-linear nature of soil and effects like liquefaction and cyclic mobility require also non-linear constitutive relations. Such advanced constitutive relations for soil under earthquake like excitation have to be validated by laboratory tests. These laboratory tests can be performed by element like tests (cyclic triaxial tests, Resonant-Column tests), with 1g shake-box tests, or centrifuge tests at higher g-levels.

With cyclic triaxial test it is not possible to reproduce shear-waves, as there is no rotation of the principal axes of strain. In Resonant-Column tests the specimen is normally brought to a resonance state. So real shear-wave propagation can only be investigated in shake-box tests. The commonly performed centrifuge tests at high g-levels have several disadvantages; however stress states like in-situ can be reproduced. This implies the use of scaling-laws which have to be fulfilled for all relevant parameters. Imagine a centrifuge test at 50g on a middle grained quartz sand (let’s say $d_{50} = 0.5$mm) in model scale, this would give a gravel with $d_{50} = 25$mm in prototype scale. Additionally care has to be taken on the

¹ Institute of Soil Mechanics and Rock Mechanics, University of Karlsruhe (TH), Engler-Bunte-Ring 14, 76131 Karlsruhe, Germany, Email: Holger.Wienbroer@ibf.uni-karlsruhe.de
scaling of viscosity of the pore fluid, the stiffness of the boundary, etc... Therefore a different concept was chosen for the 1-g shake-box presented here. The test arrangement consists of a 1D-shake-table, a laminar shake-box attached and instrumentation with data acquisition.

![Figure 1. Principle scheme of the wave propagation](image)

**SHAKE-TABLE**

The shake-table uses a one-dimensional closed loop servo controlled hydraulic drive which generates the excitation movement for the shake-box. It can be freely chosen whether a harmonic (sinusoidal) signal from a wave generator is used or random or earthquake like signal is implied. The signals only have to stay within certain limits. The input values are given in terms of displacement time amplitude or displacement time history and frequency. For frequencies lower than 0.3Hz the limiting factor is the maximum stroke of the double-acting hydraulic cylinder, which is \( u_0 = \pm 120\text{mm} \). In the frequency range between 0.3 and 10Hz the limit is given by the maximum velocity of the cylinder \( (v_0 = 0.19\text{m/s}, v_0 = u_0 \cdot \omega) \) given by the maximum oil flow rate. And for frequencies higher than 10Hz the maximum supply pressure limits the maximum acceleration amplitude \( (a_0 = 1.1g, a_0 = u_0 \cdot \omega^2) \). The maximum frequency lies in the range of about 30Hz. The asymptotic boundaries of the hydraulic drive are illustrated in Fig. 2. For the controlled real system the curve is smooth and lies beneath the asymptotic curve. Close to this line the reduction of amplitude and angular phase shift between demanded and performed movement increases.

![Figure 2. Limits of amplitude for the possible frequency range](image)
LAMINAR SHAKE-BOX

In order to avoid the difficulties in the usage of scaling laws the following concept was applied. For the validation of constitutive models like Hypoplasticity it is not necessary to bring a prototype case to model scale via scaling laws. Therefore it is only needed to have a well defined experimental setup for a simulation. This means that the state of the soil (stress state and void ratio) and state of the boundary condition have to be known at the beginning and for any time during the test. The changes have to be within the measurable range and resolution of the transducers.

In a first step the size of the shake-box has to be chosen. The most important geometrical value to be fixed is the height of the box – two aspects have to be considered here. First a certain pressure level in the soil is needed so that advanced constitutive equations fit the requirements. Second - to observe wave propagation through the soil column a major part of the wave length must fit into the column. The wave length of the shear-wave $\lambda_s$ is given by

$$\lambda_s = \frac{c_s}{f},$$

with its velocity $c_s$ and the frequency $f$. The shear-wave velocity depends on the shear modulus $G$ and the density $\rho$:

$$c_s = \sqrt{\frac{G}{\rho}}.$$  \hspace{1cm} (2)

For fully saturated quartz sand the density $\rho$ is equal to the saturation density $\rho_r$ and can be calculated via:

$$\rho = \frac{1}{1+e} \cdot \rho_s + \frac{e}{1+e} \cdot \rho_w,$$  \hspace{1cm} (3)

with the density of the grains $\rho_s$ ($\rho_s = 2.65 \text{g/cm}^3$), the density of water $\rho_w = 1.0 \text{g/cm}^3$ and the void ratio $e$. It will be shown later that the specimens of so called “Karlsruhe Sand” could be prepared with a specific density $I_D$ of about 40%. With the limit void ratios $e_{\text{max}} = 0.858$ and $e_{\text{min}} = 0.578$ and

$$I_D = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}},$$  \hspace{1cm} (4)

follows a void ratio $e = 0.746$, and with Eq. 3 a saturation density $\rho_r = 1.95 \text{g/cm}^3$ (up to $2.05 \text{g/cm}^3$ for $e_{\text{min}}$). The maximum shear modulus $G_{\text{max}}$ can be estimated with the following empirical relation for subangular grains (Hardin and Richart, 1963):

$$G_{\text{max}} = \frac{3230 \cdot (2.97 - e)^2}{1 + e} \cdot \sqrt{p'}. $$  \hspace{1cm} (5)

For technical reasons (total soil mass, functional location structure, feasibility) the shake-box height is limited to about two meters. Assuming a $2.0 \text{m}$ soil body and a specific mean density of 40% (mean saturation density $\rho_r = 1.95 \text{g/cm}^3$) a effective mean pressure $p'$ for one meter depth ($z = 1 \text{m}$) can be calculated with

$$p' = \frac{1 + 2 \cdot K_0}{3} \cdot (\rho_r - \rho_w) \cdot g \cdot z.$$  \hspace{1cm} (6)
and

\[ K_0 = 1 - \sin \varphi. \] (7)

The earth pressure coefficient \( K_0 \) is assumed to be 0.5 which implies a friction angle of \( \varphi = 30^\circ \) (\( \varphi = 30.1^\circ \) for “Karlsruhe Sand”). This results in an effective mean pressure of \( p' = 6.3 \text{kPa} \) (Eq. 6), further a maximum shear modulus after Eq. 5 of \( G_{\text{max}} = 10 \text{MPa} \) and a shear wave velocity \( c_s \) of about 72m/s for shear strains \( \leq 10^{-6} \). With this simplified elasto-dynamical approach the first natural mode (~\( \lambda/4 \)) is found at \( f = 9 \text{Hz} \), which lies inside the possible frequency range of the shake-table. This was one of the design criteria and therefore the height of the box was chosen to be 2.1m. It can be shown that the length and width of the box are of minor importance to the vibration behaviour (Wienbroer, 2003). From practical points of view the length in the direction of movement is 80cm and perpendicular the width is 60cm (soil volume of 1.01m³). The mass of the specimen is about 1500kg for medium dense dry sand. For tests with fine grained material like silt and clay it is not feasible to fill the full setup to a defined state. In this case the height of the box can be reduced to one third (0.7m).

Care has to be taken also on smooth boundaries in the deflected direction of the box, which should not constrain the wave propagation. Ordinary shake-boxes are equipped with a stack of frames with rigid walls supporting the soil sample. This setup might be feasible for n-g centrifuge tests with small differential movement of the frames. If the movements get to big one gets locally larger shear deformations at the connections of frames than in the rest of the soil body. In order to reduce the influence of these fixed walls of each frame, two oppositely placed walls (perpendicular to direction of excitation) of frames are allowed to be rotated. This leads to a smooth boundary.

Using this system for 1D-wave-propagation leads to some other requirements. The “shear” stiffness of the frame construction is normally assumed to be the same as the soil stiffness – this approach is often used in centrifuge tests. As we want to observe a large variation of soil stiffness up to liquefaction the “shear” stiffness and inertia of the pivoted frames is minimized. This also implies that the dead load of the frames has to be compensated, which is done via four “soft” springs (cf. Fig. 4 right). To minimize the influence of mass of the frames on the oscillation properties of the system their mass is reduced to be below 20% of the soil mass per frame. For the whole setup with 2.1m height the construction consists of 24 frames or 8 frames for the small setup. The soil sample inside the shake-box is surrounded by a waterproof rubber membrane.
MEASUREMENT EQUIPMENT

The horizontal deflection of each frame or lamella is measured by a displacement transducer (DT); DT 1–8 for small setup and DT 1-24 for full setup, Fig. 5. The transducer bodies are mounted on a fixed measurement-rig. The sliders are connected to the frames. An additional transducer (DT table) is connected to the moving table. Relative displacements have to be calculated from the recorded data. The measurement range of the “Novotechnik” transducers used is 300mm with a resolution / accuracy of 30µm. A direct relative measurement from frame to frame is not feasible because of the transducer size and the wiring. All other quantities like velocity, acceleration and strain must be derived from the measured displacements. Conventional piezoelectric acceleration transducers could capture the relative values directly but low frequencies they are inappropriate. Geophones are also not applicable for similar reasons.

The development of pore water pressure inside the specimen is measured in four different heights with four pore pressure transducers (PPT). The measurement range of these relative pressure transducers is 0 – 40kPa. The transducers are mounted outside the box and connected to the pore fluid via stainless steel tubes (outer diameter 4mm, inner diameter 2mm). The tubes end at heights of \( z = 0.0, 0.4, 0.8 \) and 1.2m. The upper ends are equipped with small filter stones (cf. Fig. 5). The density of the fluid or the fluidized soil above the filter stone dictates the measured pressure. All tubes are installed perpendicular to the direction of movement to avoid pressure changes due to inertial forces of the water in the tubes. Advantages of this arrangement are the ensured absence of air in the measurement system, no subsidence or ascension of the transducers during testing and that all transducers show the same pore water pressure as they are mounted at the same geodetic height. Lateral infiltration along the cables as it occurs with immersed transducers is avoided.

The settlement of the soil column is measured with two separate systems. Two laser displacement meters (LDM) with 50mm measurement range and 10µm resolution allow the continuous registration of the surface settlement and identify rocking modes. These LDM’s acquire the distance between the LDM and a small elevated target plate placed on the soil surface. Large settlements can exceed the measurement range or in the case of liquefaction the target plate sinks, in both cases the information about the settlement during one tests gets lost. An additional non contact ultra-sonic displacement meter (SDM) with a measurement range of 240mm, resolution 1mm, is used. It allows the registration of the total...
settlement of the soil or water surface during all series of tests. Measuring the height of the soil column requires the extraction of water from the soil surface after each test. The combination of LDM and SDM assures an at least good global and a good local accuracy for the displacement of the soil surface.

The data acquisition is done with a “Hottinger MGCplus” system. The sampling is done simultaneously for all transducers with a sampling rate of 600Hz. Anti-aliasing filters (40Hz and Bessel type) with constant group delay avoid dispersion.

The test series presented here only refer to tests with saturated sand in the full setup (24 frames). Tested material and sample preparation are briefly described. From the huge amount of data produced by the tests only some selected tests are discussed in detail. The settlement behaviour, liquefaction and the characteristics for very dense state of soil are in focus.

**Material**

The material used in the tests was the already mentioned “Karlsruhe Sand” which is an artificial sand-mixture with a mineralogical composition of 82% quartz, 15% feldspar and 3% calcite. The grain shape can be characterised as subangular. The limit void ratios are $e_{\text{min}} = 0.578$ and $e_{\text{max}} = 0.858$ according to the German standard DIN 18126. The critical friction angle is $\phi_c = 30.1^\circ$. Fig. 6 shows the grain size distribution which gives $d_{50} = 0.60\text{mm}$ (medium grained) and the coefficient of uniformity $C_u = d_{60}/d_{10} \approx 2$. 
Sample Preparation

To prepare a sample of saturated sand, the box first has to be fixed in the upright position by a supporting construction (pivoted lamellas vertical). Then it is filled with water up to about 1.9m - the tubes for the pore pressure measurement were installed and de-aerated before. The sand is pluviated on the water surface from a sieve box which is filled from a container. The sieve box is moved back and forth during emptying the container evenly, to achieve a relatively plane sand surface below the water. The sand is brought in with 6 strokes (total mass of dry sand: 1500kg). During this procedure the water level was kept constant (surplus water was sucked off with vacuum support) so only the sedimentation depth varies from 2.0 to 0.0m. Disadvantage of this method is a light decomposition of the sand during the sedimentation process. At the end the soil surface is planated, the compensation springs are installed and the supporting construction can be removed. Relative densities between 39 and 47% could be achieved with this installation procedure.

Testing Program

The tests were performed using a harmonic (sinusoidal) excitation signal. In a first step the frequency was kept constant (e.g. 1Hz) and the amplitude was slowly increased. 50 cycles were performed with certain amplitude and repeated 3 times, then the amplitude was increased – again 3 times 50 cycles etc... Fifty cycles were chosen to have a sufficient number of cycles to reach liquefaction. The three times repetition was done in order to reach an almost stationary state for the same excitation. An exemplary test report of a complete series is shown in Tab. 1.
### Table 1. Exemplary test report of a series of tests for the sample KAS_7xx

<table>
<thead>
<tr>
<th>Test number</th>
<th>frequency $f$ [Hz]</th>
<th>amplitude $u_0$ [mm]</th>
<th>$v_{max}$ [mm/s]</th>
<th>SDM - measurement before / after test $s_{SDM}$ [mm]</th>
<th>specific density before test $I_d$ [%]</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>KAS_701</td>
<td>1.0</td>
<td>5.0</td>
<td>31.4</td>
<td>60 60</td>
<td>47.3</td>
<td>decay of settlement</td>
</tr>
<tr>
<td>KAS_702</td>
<td>1.0</td>
<td>10.0</td>
<td>62.8</td>
<td>60 60</td>
<td>47.3</td>
<td>decay of settlement</td>
</tr>
<tr>
<td>KAS_703</td>
<td>1.0</td>
<td>15.0</td>
<td>94.2</td>
<td>67 160</td>
<td>49.4</td>
<td>1. liquefaction (partial)</td>
</tr>
<tr>
<td>KAS_707</td>
<td>1.0</td>
<td>20.0</td>
<td>125.7</td>
<td>169 171</td>
<td>79.7</td>
<td>decay of settlement</td>
</tr>
<tr>
<td>KAS_708</td>
<td>1.0</td>
<td>30.0</td>
<td>188.5</td>
<td>174 231</td>
<td>81.2</td>
<td>2. liquefaction (partial)</td>
</tr>
<tr>
<td>KAS_709</td>
<td>1.0</td>
<td>40.0</td>
<td>251.3</td>
<td>243 244</td>
<td>101.7</td>
<td>decay of settlement</td>
</tr>
<tr>
<td>KAS_710</td>
<td>2.0</td>
<td>20.0</td>
<td>251.3</td>
<td>244 250</td>
<td>102</td>
<td>delatancy/contractancy</td>
</tr>
<tr>
<td>KAS_711</td>
<td>2.0</td>
<td></td>
<td></td>
<td>300 -</td>
<td>118</td>
<td>densest state</td>
</tr>
</tbody>
</table>

### Settlement

Fig. 7 shows the entire settlement measurements (filtered with a moving average over one cycle) for two samples, the dashed black lines mark the drop-out of the LDM – measurements. The single measurements of each test were connected by the SDM values. For stages with constant amplitude the settlement rate decreases. Even after a liquefaction and reconsolidation in the first 50 cycles the next packets of cycles show stabilizing settlements. If the amplitude is increased the settlement rate increases again until the densest state is achieved.

Both samples show similar behaviour. It seems that the first liquefaction (large settlements in Fig. 7) occurs at different times for the two samples. Essential for the occurrence of liquefaction is the actual density of the specimen before the test. The sample 6xx was in looser state in the beginning and the liquefaction occurred later (first test of the fourth packet of cycles). The second liquefaction occurred at the same test conditions for both samples (almost same density).
A closer inspection can be taken with the tests KAS_704, 5 and 6 (3 times 50 cycles, \( u_0 = 10\) mm, Fig. 8). The settlement rate decreases and the pore water pressure build up gets smaller from test to test. For the first test of the series the pore pressure increase is stronger at the lowest transducer than for the higher ones. In last test of the series the increase is almost the same for all transducers.
Liquefaction

The first liquefaction occurred during test KAS_707 ($u_0 = 15\text{mm}$). Pore water pressure and settlement increase slowly (Fig. 10). After about 12sec the upper part of the soil column liquefies (about 0.8m above base level). Afterwards the excess pore water pressure decays from bottom to top, e.g. the soil reconsolidates. After 22sec the last PPT ($z = 1.2\text{m}$) is reached. From the pore pressure measurements the effective stress at the filter-stone level can be estimated (cf. Fig. 9). The current height of the soil column at the time of liquefaction was 1.975m, which results in a mean void ratio $e = 0.643$. With this void ratio a mean effective unit weight of $\gamma' = 9.74\text{kN/m}^3$ can be calculated and further the approximate effective stress $\sigma'$ at the PPT levels can be estimated (neglecting the density distribution of height). In Fig. 10 the pore water pressure $p$ increases until the total stress line is reached. With the concept of effective stress

$$\sigma' = \sigma - p$$

follows that the effective stress $\sigma'$ vanishes and the soil is liquefied. Total liquefaction occurs approximately above $z = 0.8\text{m}$.

Fig. 11 shows the contour of the shake-box and the derived shear-strains over the height for different times (the times of maximum shift between table and uppermost frame were chosen). In each case the ranges are marked, for which the drawn mode is representative. In the first 15 seconds the shear strain increases evenly over the box height. For the moment the liquefaction (12sec) the maximum shear deformation arises (up to $\gamma = 4.5\%$) in the centre of the box. The ongoing reconsolidation from bottom to top after liquefaction can be seen in the maximum in shear strain moving upward. Simultaneously the pore pressure decreases gradually. This changes stiffness and wave length as well as the mode of vibration. From 12 – 30sec the upper part of the box moves in opposite phase referred to the table and after 40sec they move in phase. Reconsolidation ends at 45sec and rigid body movement occurs (very small shear strain).
The phenomena of liquefaction and reconsolidation can also clearly be shown in test KAS_713 (Fig. 12). The specific density at the beginning of test was about 80% (dense). Due to the stronger excitation \( u_0 = 30\text{mm} \) liquefaction could again be achieved (after 14 sec). The shear deformations were larger (up to 10%).
Considerably smaller magnitudes of shear strain in the upper part of the soil column have been observed. This was the region where liquefaction occurred for the first time in test KAS_707 which may result in a locally stronger densification. In the second test reaching liquefaction (KAS_713) the gradually looser material beneath this densified area liquefies (cf. Fig. 11 and 12).

![Diagram showing shear strain and contour distribution](image)

Figure 12. Pore water pressure and settlement over time and contour and shear strain distribution over height at different times for the test KAS_713

**Delatancy and Contractancy**

For a very dense state of the soil ($I_d \approx 100\%$) delatancy and contractancy could be seen in the height measurement and the pore water pressures. Both PPT and settlement signal show a double frequency, cf. Fig. 13. The uplift of the soil surface is connected with negative pore water pressure (suction) and colour change of the sand surface (viz. brighter when dry). A mean pore water pressure increase also occurs in dense state (contractancy or compacting tendency presumed).
Resonance of the dense specimen was calculated on measured magnitudes of $u_{\text{table}}$ (constant small amplitude) and $u_{DT20}$ (current position of the soil surface at lamella 20) for increasing frequency. The amplification in Fig. 14 denotes the relation $u_{DT20} / u_{\text{table}}$. So the system is in a resonant state for a frequency of about 6Hz.

Figure 13. Pore water pressure and settlement over time for the test KAS_717

Figure 14. Resonance for the densest state of soil
CONCLUSIONS

The shake-box tests have shown that observability requires slowly varying states of soil. A careful choice and a moderate increase of excitation is fundamental to obtain slowly varying stages. The tests performed show the ability to achieve this behaviour; this implies that the basic requirements for a numerical simulation are given. Benefits of this method are: Expected effects like settlements of specimen with ongoing compaction (medium dense to dense) accompanied by pore water pressure build up, emission of water and a flow of water have been observed. Nevertheless the pore water pressure could be measured reliably as no obvious lateral infiltration could be detected. Also observable was cyclic mobility characterized with pore water pressure changes at finite displacements, but transient and inhomogeneous over the height of the sample. Phases with very low effective stress but constant pore water pressure could be observed. Dense specimens also show negative pore water pressure resulting from dilatancy.

Only due to the slowly varying processes these behaviour can be tentatively simulated with the existing formulations. Dynamic numerical simulations of the tests considering all effects require a formulation including consolidation and water flow. Until now the emission of water or flow of water can not be simulated with the constitutive models being used. Usually only fully drained or undrained conditions can be simulated, but the tests show stages comparable with these conditions. Transient changes in the modes of vibration have been simulated for undrained conditions (Rebstock et al., 2007).

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REFERENCES